

Sparse Gaussian Process-Based Strategies for Two-Layer Model Predictive Control in Autonomous Vehicle Drifting

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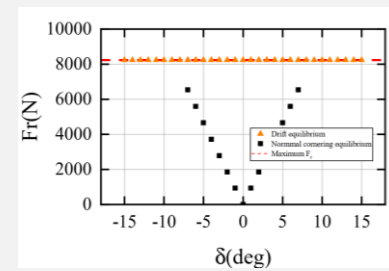
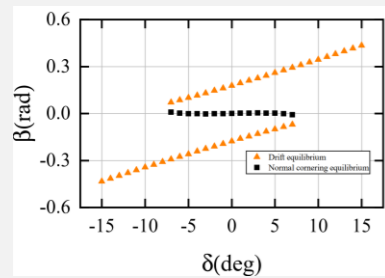
Introduction

To address the issue of **simultaneously tracking trajectories and drifting** while **accounting for mismatches** in the vehicle model, we propose a **two-layer model predictive controller based on sparse Gaussian strategies**.

Our Contributions

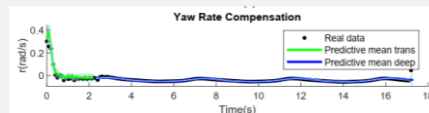
- A Two-Layer Model Predictive Controller (**TMPC**) has been proposed, which shows a **43% improvement** in tracking performance **compared to three state-of-the-art drift controllers** when tracking trajectories with varying curvature.
- The sparse variational Gaussian process model is introduced to learn the model error, which **reduces the average lateral error by 72% under model mismatch conditions**. Additionally, it shows **twice the tracking performance of FITC-based MPC** and is **ten times faster in computation time** compared to **fully GP-based MPC** [1].

Sparse Gaussian Strategies



The **critical slip angle** and **steering angle** are used as indicators to distinguish between **transit drifting** and **deep drifting**.

$$d_{VFE} = \begin{cases} d_V^1 & \text{if } |\alpha_r| \geq \alpha_{sl} \text{ and } \delta \leq \delta_{sl} \\ d_V^2 & \text{else} \end{cases}$$



Two local sparse Gaussian models are established according to the indicators. The green line is the transit drifting, and the blue is the deep drifting.

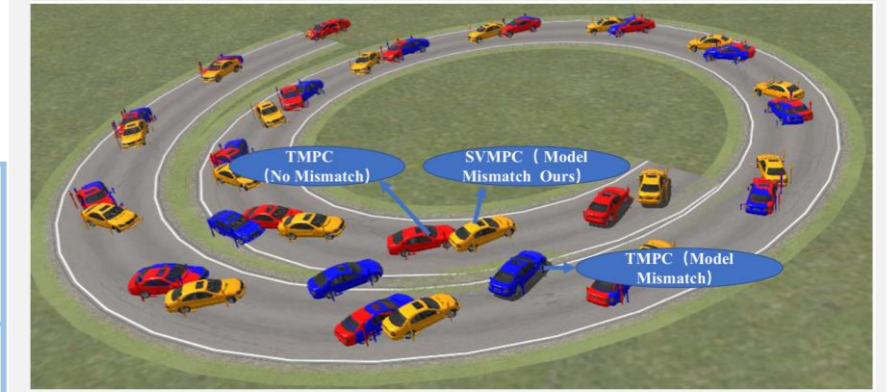
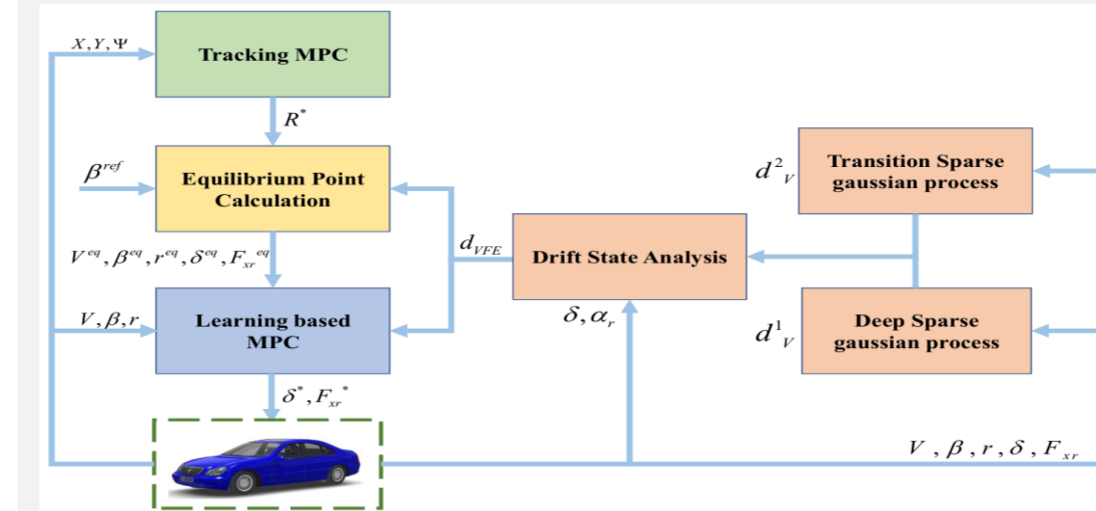
$$0 = f_v(x^l, u^l) + d_{VFE}(x^l, u^l) / T \quad x_{k+1}^l = f_d(x_k^l, u_k^l) + d_{VFE}(x_k^l, u_k^l)$$

Compensate for the **equilibrium calculation error** and the **low-level controller model error**.

Reference

- [1] Hewing L, Kabzan J, Zeilinger M N. Cautious model predictive control using gaussian process regression[J]. IEEE Transactions on Control Systems Technology, 2019, 28(6): 2736-2743.
[2] Goh J Y, Goel T, Christian Gerdes J. Toward automated vehicle control beyond the stability limits: drifting along a general path[J]. Journal of Dynamic Systems, Measurement, and Control, 2020, 142(2): 021004.
[3] Dong H, Yu H, Xi J. Real-time model predictive control for simultaneous drift and trajectory tracking of autonomous vehicles[C]//2022 6th CAA International Conference on Vehicular Control and Intelligence (CVCI). IEEE, 2022: 1-6.
[4] Chen G, Zhao X, Gao Z, et al. Dynamic drifting control for general path tracking of autonomous vehicles[J]. IEEE Transactions on Intelligent Vehicles, 2023, 8(3): 2527-2537.

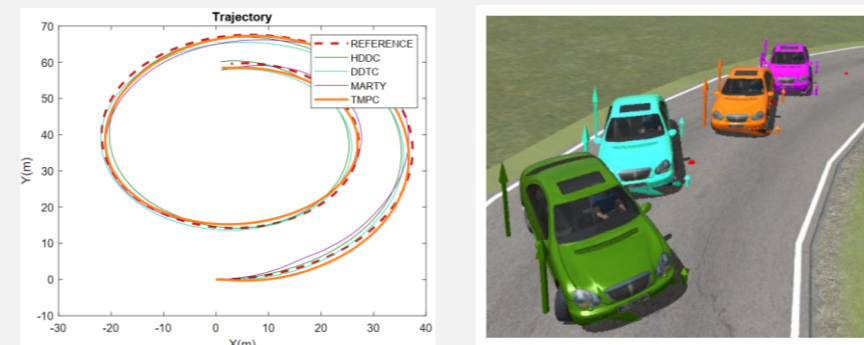
Architecture Description



Two-layer model predictive controller structure: The upper layer calculates the drift equilibrium point for trajectory tracking, while the lower layer tracks the setpoint to maintain the drift. Based on whether the vehicle has entered a deep drift state, different local Gaussians are selected for error compensations.

Comparison with State-of-the-Art Drift Controllers

Our approach is compared against **three state-of-the-art drift controllers**: 1) the unconstrained nonlinear controller **MARTY** [2], 2) the MPC controller **DDTC** [3], which simultaneously manages both drifting and trajectory tracking, 3) the **MPC-HDDC** controller [4], which combines nonlinear mapping with MPC based on the MARTY framework.



The green car represents HDDC, the blue car represents DDTC, the **yellow car** represents **TMPC (ours)**, and the purple car represents MARTY.

Controller	Avg. e(m)	Avg. e Reduction (%)	Max. e(m)	Computation time(ms)
MARTY	0.9598	54.78	1.9221	0.6759
MPC-HDDC	0.7679	43.49	1.8092	3.2329
DDTC	0.8888	51.14	1.6743	2.7649
TMPC (Ours)	0.4342	-	1.5481	9.2895

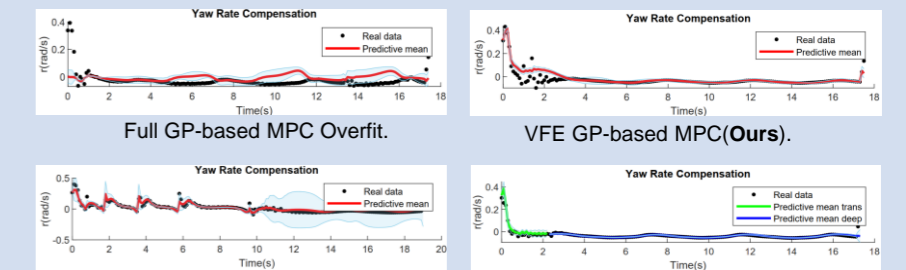
The table indicates that by decoupling the drifting and tracking problems, the **TMPC(ours)** achieved a **55%, 43%, and 51% reduction in average lateral error** compared to **MARTY, MPC-HDDC, and DDTC**, respectively, demonstrating better trajectory tracking capability.

Conclusion

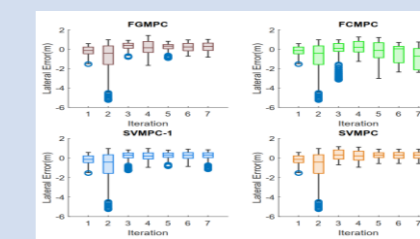
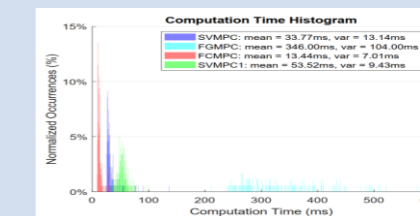
Decoupling drift and trajectory tracking in a two-layer MPC framework effectively balances both objectives. The VFE-based MPC demonstrates superior tracking and learning capabilities compared to the FITC-based MPC. Furthermore, the prediction accuracy can be improved by categorizing drift into transit drift and deep drift and establishing corresponding sparse Gaussian models.

Comparison with GP-based MPC

We introduced a 2% loss in the friction coefficient to compare the model error learning capabilities of VFE GP-based MPC(**Ours**), FITC GP-based MPC, and full GP-based MPC.



FITC GP based MPC Underfit.



TWO VFE GP-based MPC(Ours).

FITC GP-based MPC suffers from **underfitting** due to prediction variance estimation. **Full GP-based MPC** shows good prediction in some learning laps but **overfits towards the end**. **VFE-based MPC** demonstrates good prediction accuracy and is **ten times faster in computation time** compared to the full GP. **Two VFE GP-based MPC performs best**, as it has **more knowledge about when the car enters deep drifting**.

After learning five laps, the tracking performance of the **Two VFE GP-based MPC (yellow color)** is the best, as shown in the boxplot.